Chapter 13 Notes Surface Areas and Volumes

Exercise 13.5

Q.1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.

Sol. Since, diameter of the cylinder = 10 cm

$\therefore$ Radius of the cylinder (r) = $\frac{10}{2}$ cm = 5 cm

$\Rightarrow$ Length of wire in completely one round
$2\pi r = 2 \times 3.14 \times 5$ cm = 31.4 cm

$\because$ Diameter of wire = 3 mm = $\frac{3}{10}$ cm

$\therefore$ The thickness of cylinder covered in one round = $\frac{3}{10}$ m

$\Rightarrow$ Number of rounds (turns) of the wire to cover 12 cm

$$= \frac{12}{3/10} = 12 \times \frac{10}{3} = 40$$

$\therefore$ Length of wire required to cover the whole surface = Length of wire required to complete 40 rounds

= 40 × 31.4 cm = 1256 cm

Now, radius of the wire = $\frac{3}{2}$ mm = $\frac{3}{20}$ cm

$\therefore$ Volume of wire = $\pi r^2 l$

$$= \frac{22}{7} \times \frac{3}{20} \times \frac{3}{20} \times 1256 \text{ cm}^3$$

$\because$ Density of wire = 8.88 gm/cm^3

$\therefore$ Weight of the wire = [Volume of the wire] × Density

$$= \left[\frac{22}{7} \times \frac{3}{20} \times \frac{3}{20} \times 1256\right] \times 8.88 \text{ gm} = \frac{22}{7} \times \frac{3}{20} \times \frac{3}{20} \times 1256 \times \frac{888}{100} \text{ gm}$$

= 788 g (approx.)

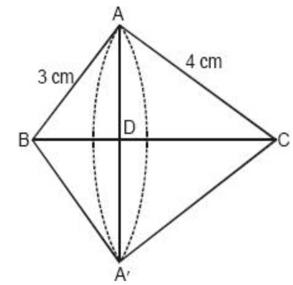
Q.2. A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. Choose value of π as found appropriate.

Sol. Let us consider the rt ΔBAC, rt. angled at A such that

AB = 3 cm, AC = 4 cm

$\therefore$ Hypotenuse BC = $\sqrt{3^2+4^2} = 5$ cm

Obviously, we have obtained two cones on the same base AA' such that the radius = DA or DA'



Now, $\frac{AD}{CA} = \frac{AB}{CB}$

$\Rightarrow \frac{AD}{4} = \frac{3}{5} \Rightarrow AD = \frac{3}{5} \times 4 = \frac{12}{6}$cm

Also, $\frac{DB}{AB} = \frac{AB}{CB}$

$\Rightarrow \frac{DB}{3} = \frac{3}{5} \Rightarrow DB = \frac{3 \times 3}{5} = \frac{9}{5}$cm

Since, CD = BC – DB $\Rightarrow$ CD = $5 - \frac{9}{5} = \frac{16}{5}$ cm

Now, volume of the double cone

$$= \left[\frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \frac{9}{5} + \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \times \frac{16}{5}\right] cm^3 = \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \left[\frac{9}{5} + \frac{16}{5}\right] cm^3$$

$$= \frac{1}{3}\pi \times \frac{144}{25} \times 5\ cm^3 = \frac{1}{3} \times \frac{314}{100} \times \frac{144}{25} \times 5\ cm^3 = 30.14\ cm^3$$

Surface area of the double cone

$$= \left(\pi \times \frac{12}{5} \times 3\right) + \left(\pi \times \frac{12}{5} \times 4\right) cm^2 = \pi \times \frac{12}{5}[3+4] cm^2$$

$$= \frac{314}{100} \times \frac{12}{5} \times 7\ cm^2 = 52.75\ cm^2$$

Q.3. A cistern, internally measuring 150 cm × 120 cm × 100 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without ocerflowing the water, each being 22.5 cm × 7.5 cm × 6.5 cm?

Sol. $\because$ Dimensions of the cistern are 150 cm, 120 cm and 110 cm.

$\therefore$ Volume of the cistern = 150 × 120 × 110 cm^3 = 1980000 cm^3

Volume of water contained in the cistern = 129600 cm^3

$\therefore$ Free space (volume) which is not filled with water

= 1980000 – 129600 cm^3 = 1850400 cm^3

Now, Volume of one brick = 22.5 × 7.5 × 6.5 cm³

$$= = \frac{225}{10} \times \frac{75}{10} \times \frac{65}{10} cm^2 = 1096.875\, cm^3$$

$\therefore$ Volume of wate absorbed by one brick = $\frac{l}{17}(1096.872)\, cm^3$

Let 'n' bricks can be put in the cistern.

$\therefore$ Volume of water absorbed by 'n' bricks = $\frac{n}{17}(1096.875)$

$\therefore$ [Volume occupied by 'n' bricks] = [(free space in the cistern) + (volume of water absorbed by n-bricks)]

$$\Rightarrow [n \times (1096.875)] = [1850400 + \frac{n}{17}(1096.875)]$$

$$\Rightarrow \left(n - \frac{n}{17}\right) \times 1096.875 = 1850400$$

$$\Rightarrow \frac{16}{17} n = \frac{1850400}{1096.875}$$

$$\Rightarrow n = \frac{1850400 \times 1000}{1096875} \times \frac{17}{16} = 1792.4102 \approx 1792$$

Thus, 1792 bricks can be put in the cistern.

Q.4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 97280 km², show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Sol. Volume of three rivers = 3 {(Surface area of a river) × Depth}

$$= 3\left\{\left(1072\, km \times \frac{75}{1000} km\right) \times \frac{3}{1000} km\right\} = 3\left\{\frac{241200}{1000000} km^3\right\}$$

$$= \frac{723600}{1000000} km^3 = 0.7236\, km^3$$

Volume of rainfall = (surface area) × (height of rainfall)

$$= 97280 \times \frac{10}{100 \times 1000} km^3$$

$$= \frac{9728}{1000} km^3 = 9.725\, km^3$$

Since, $0.7236\, km^3 \neq 9.728\, km^3$

$\therefore$ The additional water in the three rivers is not equivalent to the rainfall.

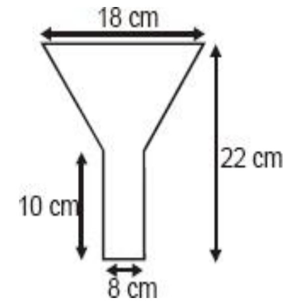
Q.5. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig.).

Sol. We have, for the cylindrical part

Diameter = 8 cm

$\Rightarrow$ Radius (r) = 4 cm

Height = 10 cm



$\Rightarrow$ Curved Surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 4 \times 10\ cm^2 = \frac{22}{7} \times 80\ cm^2$

For the frustum: $r_1 = \frac{18}{2}$ cm = 9 cm

and $r_2 = \frac{8\ cm}{2}$ cm = 4 cm

Height (H) = 22 – 10 = 12 cm

$\therefore$ Slant height (l) $= \sqrt{H^2 + (r_1 + r_2)} = \sqrt{12^2 + (9-4)^2}$

$= \sqrt{144 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$

$\therefore$ Surface area, $\pi (r_1 + r_2)\ l = \frac{22}{7} \times (4 + 9) \times 13\ cm^2$

$= \frac{22}{7} \times 13 \times 13\ cm^2 = \frac{22}{7} \times 16.9\ cm^2$

Area of tin required = [Area of the frustum] + [Area of cylindrical portion]

$= \frac{22}{7} \times 169\ cm^2 + \frac{22}{7} \times 80\ cm^2 = \frac{22}{7}\ (169 + 80)\ cm^2$

$= \frac{22}{7} \times 249\ cm^2 = \frac{5478}{7}\ cm^2 = 785\frac{4}{7}\ cm^2$

Q.6. Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

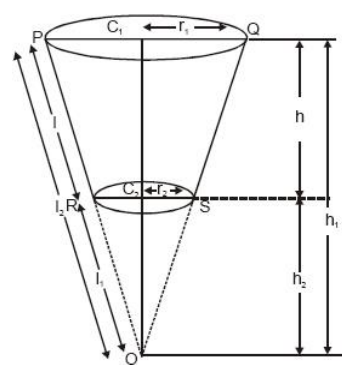
Sol. We have,

Curved surface area of the frustum PQRS

= [curved surface area of the rt circular cone OPQ] – [curved surface area of the rt circular cone ORS]

$= \pi r_1 l_1 - \pi r_2 l_2$

Now, $\Delta OC_1 Q \sim \Delta OC_2 S$



$\therefore \dfrac{OQ}{OS} = \dfrac{QC^1}{SC_2} = \dfrac{OC_1}{OC_2}$

$\Rightarrow \dfrac{l_1}{l_2} = \dfrac{r_1}{r_2} = \dfrac{h_1}{h_2}$

$\Rightarrow \dfrac{l + l_2}{l_2} = \dfrac{r_1}{r_2}$

$\Rightarrow \dfrac{l}{l_2} + 1 = \dfrac{r_1}{r_2} \Rightarrow \dfrac{l}{l_2} = \dfrac{r_1}{r_2} - 1$

$\therefore \; l = \left(\dfrac{r_1 - r_2}{r_2}\right) l_2$

Now, from (1),
curved surface area of the frustum

$= \pi r_1 \left(\dfrac{r_1}{r_2} l_2\right) - \pi r_2 l_2 = \pi l_2 \left[\dfrac{r_1^{\,2}}{r_2} - r^2\right] = \pi l_2 \left(\dfrac{r_1^{\,2} - r_2^{\,2}}{r_2}\right)$

$= \pi l_2 \left[\dfrac{(r_1 + r_2)(r_1 - r_2)}{r_2}\right] = \pi \left(\dfrac{r_1 - r_2}{r_2}\right) l_2 \times (r_1 + r_2)$

$= \pi l\,(r_1 + r_2)$

Now, the total surface area of the frustum

= (curves surface area) + (base surface area) + (top surfaces area)

$= \pi l\,(r_1 + r_2) + \pi r_2^{\,2} + \pi r_1^{\,2} = \pi(r_1 + r_2)l + \pi(r_1^{\,2} + r_2^{\,2})$

$= \pi[(r_1 + r_2)l + r^2_1 + r_2^{\,2}]$

Q.7. Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Sol. We have,

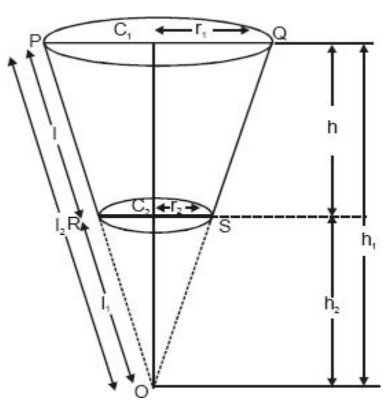
[Volume of the frustum RPQS]

= [Volume of right circular cone OPQ] – [Volume of right circular cone ORS]

$= \dfrac{1}{3}\pi r_1^{\,2} h_1 - \dfrac{1}{3}\pi r^2_{\,2} h_2$

$= \dfrac{1}{3}\pi[r_1^{\,2} h_1 - r^2_{\,2} h_2]$ ...(1)

Since $\Delta OC_1Q \sim \Delta OC_2S$



$$\therefore \frac{OQ}{OS} = \frac{QC_1}{SC_2} = \frac{OC_1}{OC_2}$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow h_1 = \left(\frac{r_1}{r_2} \times h_2\right) \qquad ...(2)$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{h + h_2}{h_2}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{h}{h_2} + 1$$

$$\Rightarrow \frac{h}{h_2} = \frac{r_1}{r_2} - 1 \Rightarrow h = \left[\frac{r_1}{r_2} - 1\right] \times h_2$$

$$\Rightarrow h = (r_1 - r_2)\frac{h_2}{r_2} \qquad ...(3)$$

From (1) and (2), we have

$$\{\text{volume of the frustum RPQS}\} = \frac{1}{3}\pi\left[r_1^{\ 2} \times \frac{r_1}{r_2} h_2 - r^2{}_2 h_2\right]$$

$$= \frac{1}{3}\pi\left[\frac{r_1^{\ 3}}{r_2} - r^2{}_2\right] h_2$$

$$= \frac{1}{3}\pi[r_1^{\ 3} - r_2^{\ 3}]\frac{h_2}{r_2}$$

$$= \frac{1}{3}\pi\left(r_1^{\ 2} + r^2{}_2 + r_1 r_2\right)\left[\left(r_1 - r_2\right)\frac{h_2}{r_2}\right]$$

$$= \frac{1}{3}\pi\left(r_1^{\ 2} + r^2{}_2 + r_1 r_2\right) h$$